

# Non-Unitary Lepton Mixing Matrix, Leptogenesis and Low Energy CP Violation

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## Abstract

It is well-known that for unflavored leptogenesis there is in general no connection between low and high energy CP violation. We stress that for a non-unitary lepton mixing matrix this may not be the case. We give an illustrative example for this connection and show that the non-standard CP phases that are induced by non-unitarity can be responsible for observable effects in neutrino oscillation experiments as well as for the generation of the baryon asymmetry of the Universe. Lepton Flavor Violation in decays such as  $\tau \rightarrow \mu\gamma$  can also be induced at an observable level. We also comment on neutrino mass limits from leptogenesis, which get barely modified in case of a non-unitary mixing matrix.

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# 1 Introduction

Hands-on beyond the Standard Model physics became reality when observations of neutrino oscillations showed that neutrino masses are non-zero. The most appealing scenario to explain the smallness of neutrino masses is the see-saw mechanism [1], which suppresses their mass scale by the presence of new heavy Majorana neutrinos. As a bonus, there is the possibility that the baryon asymmetry of the Universe is generated by the out-of-equilibrium decay of these heavy neutrinos, the leptogenesis mechanism [2]. Unfortunately, there is a “no connection” theorem [3], which states that low energy CP violation in neutrino oscillation experiments is independent from the high energy CP violation responsible for unflavored leptogenesis.

In the meanwhile, neutrino physics has entered the precision era. In the next decade there will be a plethora of new neutrino oscillation and mass-related experiments, whose purpose it is to probe the unknown parameters of the neutrino mass matrix and to determine the already known ones with high precision [4]. Soon the “standard picture” of three active neutrinos whose mixing is described by a unitary matrix can be put to the test. Here we will assume that the standard picture is incomplete, in the sense that the lepton mixing matrix deviates from being unitary. This means that the three active neutrinos  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$  are connected to the mass states  $\nu_1$ ,  $\nu_2$  and  $\nu_3$  via  $\nu_\alpha = N_{\alpha i} \nu_i$ , where  $NN^\dagger \neq \mathbb{1}$ . As usual,  $\alpha = e, \mu, \tau$  is the flavor index and  $i = 1, 2, 3$  the mass index. This possible feature has recently been discussed by several authors [5, 6, 7, 8, 9, 10]. It turns out that many theories beyond the Standard Model, which also incorporate massive neutrinos, have the capacity to induce a non-unitary PMNS matrix. There are various straightforward examples with such a net effect, for instance mixing with sterile neutrinos or supersymmetric particles, or non-standard interactions [11, 12]. Here we will discuss some peculiar implications of a non-unitary PMNS matrix. For instance, we note that the “no connection” theorem between low and high energy CP violation no longer holds. We furthermore show that the phases associated with the non-unitarity of the lepton mixing matrix can be sufficient to generate the correct amount of the baryon asymmetry and at the same time can lead to spectacular effects in neutrino oscillation experiments. As one example in which this interesting situation may be realized, we utilize a simple extension of the see-saw mechanism. As a further simple application, we show that lepton flavor violating processes such as  $\tau \rightarrow \mu \gamma$  can be generated at observable levels. Moreover, the non-unitarity of the PMNS matrix is here shown to lead to maximal values of the decay asymmetry which are larger than for a unitary mixing matrix. The small impact on neutrino mass limits is discussed.

The paper is build up as follows: in Section 2 we introduce non-unitarity of the PMNS matrix and discuss how it influences the see-saw reconstruction. Section 3 deals with non-unitarity phenomenology in neutrino oscillations, lepton flavor violation and leptogenesis. After investigating neutrino mass constraints from leptogenesis in Section 4 we conclude in Section 5.

## 2 Non-Unitarity, the See-Saw Mechanism and Leptogenesis

In the conventional see-saw framework there are Dirac and Majorana mass matrices  $m_D$  and  $M_R$  in the Lagrangian [1]

$$\mathcal{L} = \frac{1}{2} \overline{N_R} M_R N_R^c + \overline{N_R} m_D \nu_L + h.c. \quad (1)$$

Without loss of generality we will assume here that  $M_R$  is real and diagonal. The low energy mass matrix is

$$m_\nu = -m_D^T M_R^{-1} m_D. \quad (2)$$

In the charged lepton basis and the standard picture  $m_\nu$  is diagonalized by a unitary matrix as  $U^* P^* m_\nu^{\text{diag}} P^\dagger U^\dagger = m_\nu$ . The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix is  $U P$ , where  $U$  has the standard form

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (3)$$

Here  $c_{ij} = \cos \theta_{ij}$ ,  $s_{ij} = \sin \theta_{ij}$  and the Majorana phases are contained in  $P = \text{diag}(1, e^{i\alpha}, e^{i\beta})$ .

Leptogenesis is a possible consequence of the see-saw mechanism and generates the baryon asymmetry of the Universe via decays of a (usually the lightest) heavy Majorana neutrino into lepton and Higgs doublets in the early Universe [13]. One distinguishes flavored and unflavored leptogenesis. For unflavored leptogenesis, valid for  $M_1 \gtrsim 10^{11}$  GeV, the flavor of the final state leptons plays no role. Leptogenesis for lower values of  $M_1$  can be shown to depend on the flavor of the final state leptons, and is called flavored leptogenesis [14]. Here we focus on unflavored leptogenesis in which case the decay asymmetry is given by

$$\varepsilon_1 = \frac{1}{8\pi v^2} \frac{1}{(m_D m_D^\dagger)_{11}} \sum_{j=2,3} \text{Im} \left\{ (m_D m_D^\dagger)_{1j}^2 \right\} f(M_j^2/M_1^2),$$

where  $f(x) \simeq -\frac{3}{2\sqrt{x}}$  for  $x \gg 1$ , i.e., hierarchical heavy neutrinos. The baryon asymmetry of the Universe is proportional to the decay asymmetry  $\varepsilon_1$ . It is well-known that in the general case the CP violation responsible for unflavored leptogenesis bears no connection to low energy lepton mixing angles and CP phases (for analyses of low and high energy CP violation in case of flavored leptogenesis, see e.g. [15, 16]). One simple proof of this fact, which we repeat here, uses the Casas-Ibarra parametrization of the Dirac mass matrix in terms of measurable parameters and a complex and orthogonal matrix  $R$  [17]:

$$m_D = i \sqrt{M_R} R \sqrt{m_\nu^{\text{diag}}} U^\dagger. \quad (4)$$

The matrix  $R$  contains six free parameters and can be parameterized (up to reflections) as

$$R = \begin{pmatrix} c_{12}^R c_{13}^R & s_{12}^R c_{13}^R & s_{13}^R \\ -s_{12}^R c_{23}^R - c_{12}^R s_{23}^R s_{13}^R & c_{12}^R c_{23}^R - s_{12}^R s_{23}^R s_{13}^R & s_{23}^R c_{13}^R \\ s_{12}^R s_{23}^R - c_{12}^R c_{23}^R s_{13}^R & -c_{12}^R s_{23}^R - s_{12}^R c_{23}^R s_{13}^R & c_{23}^R c_{13}^R \end{pmatrix},$$

with complex angles  $\theta_{ij}^R$  and  $c_{ij}^R = \cos \theta_{ij}^R$ ,  $s_{ij}^R = \sin \theta_{ij}^R$ . The relevant quantity for leptogenesis is then

$$\begin{aligned} m_D m_D^\dagger &= \sqrt{M_R} R \sqrt{m_\nu^{\text{diag}}} \underbrace{U^\dagger U}_{=\mathbb{1}} \sqrt{m_\nu^{\text{diag}}} R^\dagger \sqrt{M_R} \\ &= \sqrt{M_R} R m_\nu^{\text{diag}} R^\dagger \sqrt{M_R} \end{aligned}$$

Hence, owing to the *assumed unitarity* of the PMNS matrix, the low energy mixing matrix elements (and in particular the CP phases) drop out of the expression for the decay asymmetry [3]. Note further that if  $R$  is real then there is no leptogenesis at all.

As given in the above expression Eq. (3),  $U$  is indeed manifestly unitary and the “no connection” theorem is valid. Here we shall assume however that the lepton mixing matrix, from now on called  $N$ , is non-unitary. It proves convenient to write in the relation  $\nu_\alpha = N_{\alpha i} \nu_i$ , which connects flavor and mass states, the non-unitary matrix  $N$  as [7]

$$N = (1 + \eta) U_0, \quad (5)$$

where  $\eta$  is hermitian (containing 6 real moduli and 3 phases) and  $U_0$  is unitary (containing 3 real moduli and 3 phases). Several observables lead to 90% C.L. bounds on  $\eta$  [6]:

$$\begin{pmatrix} |\eta_{ee}| & |\eta_{e\mu}| & |\eta_{e\tau}| \\ \cdot & |\eta_{\mu\mu}| & |\eta_{\mu\tau}| \\ \cdot & \cdot & |\eta_{\tau\tau}| \end{pmatrix} < \begin{pmatrix} 5.5 \times 10^{-3} & 3.5 \times 10^{-5} & 8.0 \times 10^{-3} \\ \cdot & 5.0 \times 10^{-3} & 5.1 \times 10^{-3} \\ \cdot & \cdot & 5.1 \times 10^{-3} \end{pmatrix}. \quad (6)$$

More importantly for our matters, the possible CP phases of the elements of  $\eta$  ( $\eta_{\alpha\beta} = |\eta_{\alpha\beta}| e^{i\phi_{\alpha\beta}}$  for  $\alpha \neq \beta$ ) are not constrained. If  $m_\nu$ , which is diagonalized by a non-unitary mixing matrix, stems from the see-saw mechanism, we have now instead of Eq. (4)

$$m_D = i \sqrt{M_R} R \sqrt{m_\nu^{\text{diag}}} N^\dagger, \quad (7)$$

and thus for leptogenesis

$$\begin{aligned} m_D m_D^\dagger &= \sqrt{M_R} R \sqrt{m_\nu^{\text{diag}}} \underbrace{N^\dagger N}_{\simeq \mathbb{1} + 2 U_0^\dagger \eta U_0 \neq \mathbb{1}} \sqrt{m_\nu^{\text{diag}}} R^\dagger \sqrt{M_R} \\ &\simeq \mathbb{1} + 2 U_0^\dagger \eta U_0 \neq \mathbb{1} \end{aligned} \quad (8)$$

We see that leptogenesis is no longer independent on the low energy phases. It is sensitive to the phases in  $U_0$  as well as to the phases in  $\eta$ . Moreover, in the expression  $\text{Im}\{(m_D m_D^\dagger)_{1j}^2\}$ , which appears in the decay asymmetry  $\varepsilon_i$ , the non-unitarity parameter appears in first order! In order to underscore the correlation between low and high energy CP violation we

will consider the case of real  $R$  from now on. For complex  $R$  the effect of the non-standard phases can be expected to be suppressed by the smallness of the  $\eta_{\alpha\beta}$ . Nevertheless, as we will show below, the non-standard CP phases included in  $\eta$  are sufficient to generate the baryon asymmetry of the Universe via the leptogenesis mechanism<sup>1</sup>. In addition, spectacular effects in neutrino oscillation experiments can be induced [7, 8].

First of all we will outline a possible see-saw extension which incorporates the situation we are after. The intrinsic non-unitarity in general see-saw scenarios has first been noted in [18]. Let us stress here that also the conventional see-saw mechanism implies that the PMNS matrix is non-unitary. However, for the natural case of heavy Majorana neutrino masses (which we require in order to have thermal leptogenesis) this effect is way too tiny to lead to any observable signature, see e.g. [10]. Note that in order to link the lepton mixing matrix to leptogenesis via the Casas-Ibarra parametrization, there should be no sizable contribution to  $m_\nu$  and leptogenesis other than the usual see-saw terms. Hence we need to decouple the source of unitary violation from these terms, but still allow some mixing of the light neutrinos with new physics, thereby creating non-unitarity in the low energy mixing matrix. Consider the see-saw mechanism extended by an additional singlet sector, leading to a  $9 \times 9$  mass matrix

$$\mathcal{L} = \frac{1}{2} \left( \overline{\nu_L^c}, \overline{N_R}, \overline{X} \right) \begin{pmatrix} 0 & m_D^T & m^T \\ m_D & M_R & 0 \\ m & 0 & M_S \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \\ X^c \end{pmatrix} + h.c.,$$

where the upper left block is the usual see-saw. We can diagonalize it with a unitary matrix  $\mathcal{U}$  defined as

$$\mathcal{U} = \begin{pmatrix} \tilde{N} & S & A \\ T & V & D \\ B & E & W \end{pmatrix}, \text{ where}$$

$$\mathcal{U}^T \begin{pmatrix} 0 & m_D^T & m^T \\ m_D & M_R & 0 \\ m & 0 & M_S \end{pmatrix} \mathcal{U} = \begin{pmatrix} m_\nu^{\text{diag}} & 0 & 0 \\ 0 & M_R & 0 \\ 0 & 0 & M_S^{\text{diag}} \end{pmatrix}.$$

The usual see-saw terms take their usual magnitudes,  $M_R \sim 10^{15}$  GeV and  $m_D \sim 10^2$  GeV. Consequently,  $S$  and  $T$  are of order  $m_D/M_R \sim 10^{-13}$ . Assume now that the rotation that eliminates the 13-entry  $m$  (corresponding to the matrices  $A$  and  $B$ ) is of order  $10^{-2}$ . This rotation introduces additional terms to the low energy neutrino mass matrix of order  $m \times 10^{-2}$  and  $M_S \times 10^{-4}$ . We need to suppress these entries with respect to  $m_D^2/M_R$ . For instance, if  $m$  is of order  $10^{-13}$  GeV and  $M_S$  of order  $10^{-15}$  GeV, it is easy to see that  $A$  and  $B$  are of order  $10^{-2}$ . These features together with the mentioned magnitudes of  $S$  and  $T$  are enough to obtain from the upper left and upper middle elements of the mass matrix:

$$m_\nu^{\text{diag}} = -\tilde{N}^T m_D^T M_R^{-1} m_D \tilde{N}. \quad (9)$$

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<sup>1</sup>Let us note that flavored leptogenesis, where in general contributions from low energy phases can be expected [15, 16], will of course receive contributions from the non-standard phases as well.

This shows that  $N = (\tilde{N}^\dagger)^{-1}$  is the leptonic mixing matrix, since there is no other sizable contribution to the mass term of the light active neutrinos.  $N$  is non-unitary, because the 11-element of  $\mathcal{U}\mathcal{U}^\dagger$  gives the constraint  $(N^\dagger)^{-1}N^{-1} + SS^\dagger + AA^\dagger = \mathbb{1}$ . Because the non-unitary contribution from  $S$  is extremely tiny, we can neglect it. Note that in this case Eq. (5) and  $(N^\dagger)^{-1}N^{-1} + SS^\dagger + AA^\dagger = \mathbb{1}$  imply  $2\eta \simeq AA^\dagger$ . The unitarity violation is therefore governed by the magnitude of the elements of the matrix  $A$ , which is of order  $10^{-2}$ , thereby possibly reaching the limits on  $\eta$  given above.

Within this situation we have sketched how to keep the usual see-saw mechanism with unmodified leptogenesis (note that  $M_R$  does not couple to the new singlets)<sup>2</sup> but to induce a sizably non-unitary lepton mixing matrix  $N$ . Note that the hierarchies in the individual mass matrices should not be too extreme in order for this mechanism to work. Nevertheless, we are satisfied that there are indeed frameworks which allow us to apply Eq. (8). Another example could be non-standard interactions mediated by higher dimensional operators which lead to non-unitarity [12]. The new heavy particles introducing these operators are supposed to not influence the see-saw formula for  $m_\nu$  and leptogenesis, which is a constraint on those scenarios. We will however continue our analysis independent of any possible underlying setup and simply analyze the consequences of Eq. (8).

### 3 Low Energy Phenomenology of Non-Unitarity and Leptogenesis

One interesting aspect of non-unitarity of the PMNS matrix lies in Lepton Flavor Violation (LFV). It is well-known that even for the unitary case massive neutrinos can induce LFV (although at unobservably small levels) in decays such as  $\alpha \rightarrow \beta \gamma$  with  $(\alpha, \beta) = (\tau, \mu)$ ,  $(\tau, e)$  or  $(\mu, e)$ . The branching ratio normalized to the leading decay in charged leptons and neutrinos is given by [19]

$$\frac{\text{BR}(\alpha \rightarrow \beta \gamma)}{\text{BR}(\alpha \rightarrow \beta \bar{\nu} \nu)} = \frac{3}{32\pi} \frac{\alpha}{\pi} \left| U_{\alpha i} U_{i\beta}^\dagger f(x_i) \right|^2, \quad (10)$$

where the light neutrino  $m_i$  masses appear in  $x_i = m_i^2/m_W^2$  ( $m_W$  being the mass of the  $W$ ) and the loop function is

$$f(x) = \frac{1}{3} \frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \ln x}{(1-x)^4}.$$

Because  $x \ll 1$  one has  $f(x) \simeq 10/3 - x$  and the first order term in Eq. (10) is absent when summed over  $i$ . This GIM suppression mechanism leading to unobservably small branching ratios proportional to  $m_i^4/m_W^4$  is a consequence of the unitarity of the PMNS matrix.

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<sup>2</sup>There is a suppressed contribution to the decay asymmetry in which two mass insertions mediated by the  $\nu_L$ - $X$ -vertex show up in the usual self-energy contribution.

Now consider unitarity violation: the formula for the branching ratio  $\alpha \rightarrow \beta \gamma$  is the same as above in Eq. (10), except that now  $U$  is to be replaced with  $N$ . Since  $N$  is not unitary the first order term from  $f(x)$  may be the leading contribution and the branching ratio is

$$\frac{\text{BR}(\alpha \rightarrow \beta \gamma)}{\text{BR}(\alpha \rightarrow \beta \bar{\nu} \nu)} \simeq \frac{100 \alpha}{96 \pi} |(NN^\dagger)_{\alpha\beta}|^2. \quad (11)$$

The fact that the experimental limit  $\text{BR}(\mu \rightarrow e \gamma) < 1.2 \cdot 10^{-11}$  [20], is almost four orders of magnitude stronger than the limits  $\text{BR}(\tau \rightarrow e \gamma) < 3.3 \cdot 10^{-8}$  or  $\text{BR}(\tau \rightarrow \mu \gamma) < 4.4 \cdot 10^{-8}$  [21] is the reason why the limit on  $|\eta_{e\mu}|$  is almost two orders of magnitude better than the ones on  $|\eta_{e\tau}|$  or  $|\eta_{\mu\tau}|$ . We consider from now on only the term  $|\eta_{\mu\tau}| e^{i\phi_{\mu\tau}}$  to illustrate our points with a simple example. The simple result is

$$\text{BR}(\tau \rightarrow \mu \gamma) \simeq \text{BR}(\tau \rightarrow \mu \bar{\nu} \nu) \frac{25 \alpha}{6 \pi} |\eta_{\mu\tau}|^2. \quad (12)$$

Note that with  $N = (1 + \eta) U_0$  the parameters in  $U_0$  do not appear in  $NN^\dagger$ . Using  $\text{BR}(\tau \rightarrow \mu \bar{\nu} \nu) = 0.1736$  and  $\text{BR}(\tau \rightarrow \mu \gamma) < 4.4 \cdot 10^{-8}$  [21] one can obtain the constraint  $|\eta_{\mu\tau}| < 5.1 \cdot 10^{-3}$  quoted above. Alternatively, if  $|\eta|$  is given, or constrained to lie in a certain range, one can predict the rate of  $\tau \rightarrow \mu \gamma$ . Indeed, rewriting the last expression gives a useful estimate:

$$\text{BR}(\tau \rightarrow \mu \gamma) \simeq 4.2 \cdot 10^{-10} \left( \frac{|\eta_{\mu\tau}|}{5 \cdot 10^{-4}} \right)^2. \quad (13)$$

It has been estimated that improvements on the experimental limits of  $\text{BR}(\tau \rightarrow \beta \gamma)$  by one to two orders of magnitude are possible [22]. Sensitivities around  $2 \cdot 10^{-9}$  correspond to values of  $|\eta_{\mu\tau}|$  around  $1.1 \cdot 10^{-3}$ .

As a next application of unitarity violation we consider effects in neutrino oscillation experiments: it is known [7, 8] that non-unitarity of the PMNS matrix can lead to CP asymmetries in neutrino oscillations, which are dramatically different from the ones in the standard case. One defines asymmetries

$$A_{\alpha\beta}^{\text{SM}} = \frac{P_{\alpha\beta}^{\text{SM}} - P_{\bar{\alpha}\bar{\beta}}^{\text{SM}}}{P_{\alpha\beta}^{\text{SM}} + P_{\bar{\alpha}\bar{\beta}}^{\text{SM}}}, \quad (14)$$

where  $P_{\alpha\beta}^{\text{SM}}$  is the standard (vacuum) oscillation probability for the channel  $\nu_\alpha \rightarrow \nu_\beta$  and  $P_{\bar{\alpha}\bar{\beta}}^{\text{SM}}$  the corresponding probability for anti-neutrinos:

$$P_{\alpha\beta}^{\text{SM}} = \left| \sum_j U_{\beta j} U_{\alpha j}^* e^{-im_j^2 L/(2E)} \right|^2 \quad (15)$$

and  $P_{\bar{\alpha}\bar{\beta}}^{\text{SM}} = P_{\alpha\beta}^{\text{SM}}(U \rightarrow U^*)$ . Survival probability asymmetries  $A_{\alpha\alpha}^{\text{SM}}$  are zero and  $A_{\alpha\beta}^{\text{SM}} = -A_{\beta\alpha}^{\text{SM}}$ . We focus from now on on one particular setup, in which we can simplify the expressions. We assume negligible matter effects and  $\Delta_{21} \ll |\Delta_{31}|$ , where  $\Delta_{ij} = \Delta m_{ij}^2 L/(2E)$

with  $\Delta m_{ij}^2 = m_i^2 - m_j^2$ . This may be realized in a high-energy, short-baseline neutrino factory, which has been discussed in the framework of non-unitarity effects in oscillations first in Ref. [7]. Following those authors we will choose as an example a muon energy  $E_\mu = 50$  GeV (hence  $\langle E_{\nu_\mu} \rangle = \frac{7}{10} E_\mu$ ) and baseline  $L = 130$  km. It was found that the  $\mu$ - $\tau$  channel is particularly powerful to detect new physics. In the unitary case and with the simplifications given above one finds for  $\sin^2 \theta_{12} = \frac{1}{3}$  and  $\sin^2 \theta_{23} = \frac{1}{2}$  that

$$A_{\mu\tau}^{\text{SM}} \simeq \frac{2\sqrt{2}}{3} \Delta_{21} |U_{e3}| \sin \delta. \quad (16)$$

Numerically, for  $\theta_{13} = 0.1$  one finds from the full expression that  $|A_{\mu\tau}^{\text{SM}}| \lesssim 7 \times 10^{-5}$ .

For a non-unitary lepton mixing matrix the oscillation probabilities are

$$P_{\alpha\beta}^{\text{NU}} = \frac{|\sum_j N_{\beta j} N_{\alpha j}^* e^{-im_j^2 L/(2E)}|^2}{(NN^\dagger)_{\alpha\alpha} (NN^\dagger)_{\beta\beta}} \equiv \frac{\hat{P}_{\alpha\beta}^{\text{NU}}}{(NN^\dagger)_{\alpha\alpha} (NN^\dagger)_{\beta\beta}}.$$

For anti-neutrinos one has  $P_{\bar{\alpha}\bar{\beta}}^{\text{NU}} = P_{\alpha\beta}^{\text{NU}}(N \rightarrow N^*)$ . As can be seen, there is just as for “normal” oscillations no sensitivity on the “Majorana phases” which appear on the right of  $U_0$ . The CP asymmetries are

$$A_{\alpha\beta}^{\text{NU}} = \frac{\hat{P}_{\alpha\beta}^{\text{NU}} - \hat{P}_{\bar{\alpha}\bar{\beta}}^{\text{NU}}}{\hat{P}_{\alpha\beta}^{\text{NU}} + \hat{P}_{\bar{\alpha}\bar{\beta}}^{\text{NU}}}. \quad (17)$$

The different  $A_{\alpha\beta}^{\text{NU}}$  are in general independent of each other, though (just as in the standard case) survival probability asymmetries  $A_{\alpha\alpha}^{\text{NU}}$  are zero and  $A_{\alpha\beta}^{\text{NU}} = -A_{\beta\alpha}^{\text{NU}}$ . Note that the asymmetries were constructed in terms of “bare” probabilities, i.e., probabilities stripped of their normalization factors  $(NN^\dagger)_{\alpha\alpha}$ .

For simplicity, we will assume from now on that  $U_0$ , which can be parameterized in analogy to Eq. (3), is characterized by  $\sin^2 \theta_{12} = \frac{1}{3}$  and  $\sin^2 \theta_{23} = \frac{1}{2}$ . This is motivated by the facts that at leading order  $U_0$  can be identified with the PMNS matrix, and that this matrix is at leading order well described [23] by tri-bimaximal mixing. The CP asymmetry in the  $\mu$ - $\tau$  sector is then

$$A_{\mu\tau}^{\text{NU}} \simeq \frac{-4|\eta_{\mu\tau}|}{\cos^2 \theta_{13}} \cot \frac{\Delta_{31}}{2} \sin \phi_{\mu\tau}. \quad (18)$$

Hence, in the experimental setup under consideration  $|A_{\mu\tau}^{\text{NU}}|$  can easily exceed the maximal value of  $|A_{\mu\tau}^{\text{SM}}| \lesssim 7 \times 10^{-5}$  given above [7, 8].

Returning to leptogenesis, the baryon asymmetry should lie in the interval  $(8.75 \pm 0.23) \times 10^{-11}$  and is given by [13]  $Y_B \simeq 1.27 \times 10^{-3} \varepsilon_1 \eta(\tilde{m}_1)$ , where  $g_* \simeq 106$ ,  $\tilde{m}_1 = (m_D m_D^\dagger)_{11}/M_1$  and  $\eta(x) \simeq 1/((8.25 \times 10^{-3} \text{ eV})/x + (x/(2 \times 10^{-4} \text{ eV}))^{1.16})$  describes the wash-out. The in general complex and orthogonal matrix  $R$  in the parametrization of  $m_D$  from Eq. (7) is here chosen real (to forbid unflavored leptogenesis in case of a unitary PMNS matrix)

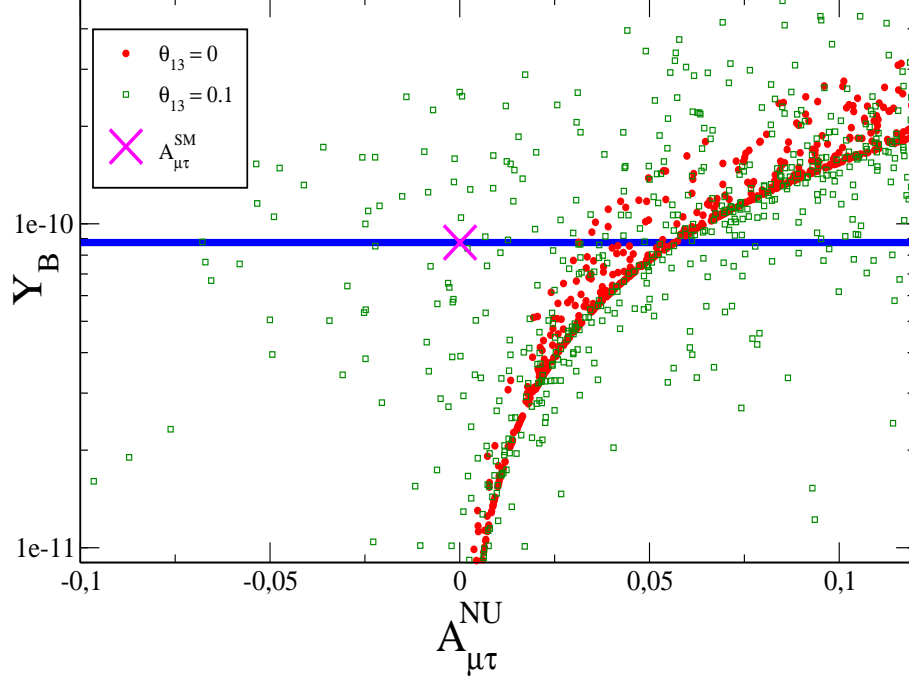


Figure 1: Scatter plot of the CP asymmetry in neutrino oscillations for a 50 GeV neutrino factory with baseline 130 km against the baryon asymmetry of the Universe. We have chosen  $\theta_{12}^R = 1.1$ ,  $\theta_{23}^R = 0.3$  and  $\theta_{13}^R = 0.8$ . The blue horizontal line is the observed value of  $Y_B$ . The maximal value for  $|A_{\mu\tau}|$  in the Standard Model is marked by a magenta cross and is  $7 \times 10^{-5}$  for  $\theta_{13} = 0.1$ .

and described by three real Euler angles  $\theta_{ij}^R$  with  $ij = 12, 13, 23$ . The light neutrino masses are normally ordered with  $m_1 = 10^{-3}$  eV,  $m_2^2 - m_1^2 = \Delta m_{\odot}^2$  and  $m_3^2 - m_1^2 = \Delta m_A^2$ , where  $\Delta m_{\odot}^2 = 7.67 \times 10^{-5}$  eV<sup>2</sup> and  $\Delta m_A^2 = 2.39 \times 10^{-3}$  eV<sup>2</sup> [23]. Neglecting  $m_1$  and  $\theta_{13}$  we find

$$\varepsilon_1 \simeq \sqrt{\frac{3}{8\pi^2}} \frac{M_1 \sqrt{m_2 m_3}}{v^2} \frac{(m_2 - m_3) c_{13}^R s_{12}^R s_{13}^R}{m_2 R_{12}^2 + m_3 R_{13}^2} |\eta_{\mu\tau}| \sin \phi_{\mu\tau}, \quad (19)$$

where  $R_{ij}$  are the elements of  $R$  and  $s_{ij}^R = \sin \theta_{ij}^R$ ,  $c_{ij}^R = \cos \theta_{ij}^R$ . Hence, just as the oscillation CP asymmetry in the  $\mu$ - $\tau$  sector in Eq. (18), and just as expected from Eq. (8), the decay asymmetry depends linearly on  $|\eta_{\mu\tau}| \sin \phi_{\mu\tau}$ . Note that  $\varepsilon_1 = 0$  for  $\eta_{\mu\tau} = 0$ . Hence, the dependence of  $\varepsilon_1$  on the “standard phase”  $\delta$  should be at most proportional to  $|\eta_{\mu\tau}| \theta_{13} \sin \delta$ , which is indeed the case. For  $M_1$ ,  $m_2$  and  $m_3$  given as above and for, say,  $\theta_{12}^R = \theta_{13}^R = \pi/4$ , the order of magnitude of the decay asymmetry is  $\varepsilon_1 \simeq -10^{-4} |\eta_{\mu\tau}| \sin \phi_{\mu\tau}$ . The wash-out parameter  $\tilde{m}_1$  is of order  $10^{-2}$  eV. This means we are in the favorable “strong wash-out regime”, in which the final baryon asymmetry has very little to no dependence on the initial conditions (i.e., the initial abundance of the heavy neutrinos) [13]. In this regime  $\eta(\tilde{m}_1)$  is of order  $10^{-2}$ , and in total we end up with

$$Y_B \sim \text{few } 10^{-9} |\eta_{\mu\tau}| \sin \phi_{\mu\tau}, \quad (20)$$

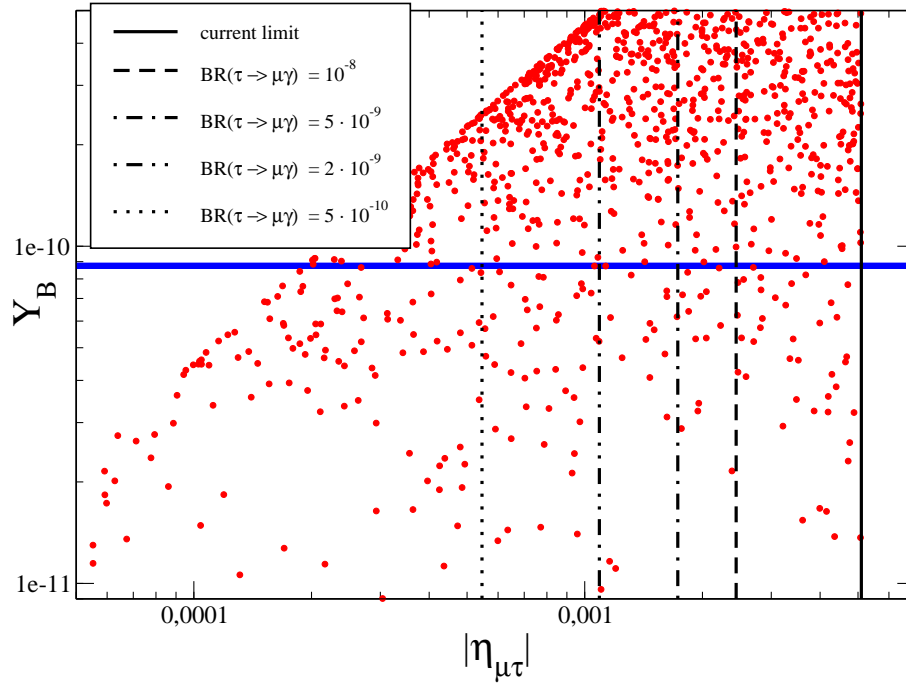


Figure 2: Scatter plot of  $|\eta_{\mu\tau}|$  against the baryon asymmetry of the Universe for the same parameters as in Fig. 1 ( $\theta_{13} = 0$ ). Certain values of  $|\eta_{\mu\tau}|$  which correspond to certain observable benchmark values of the branching ratio for the decay  $\tau \rightarrow \mu\gamma$  have also been indicated.

which for  $|\eta_{\mu\tau}|$  of order few times  $10^{-3}$  is in reasonable agreement with the observed range it should lie in. A more detailed numerical study using no approximations results in Fig. 1, which illustrates the interplay of low and high energy CP violation for a (by no means special) point in the parameter space spanned by  $R$ . We have varied  $|\eta_{\mu\tau}|$  and  $\phi_{\mu\tau}$  within their allowed ranges and took as heavy neutrino parameters  $M_1 = 2.5 \cdot 10^{12}$  GeV,  $M_2 = 2 \cdot 10^{13}$  GeV and  $M_3 = 10^{14}$  GeV. Note that  $M_1$  is chosen such that flavor effects in leptogenesis play no role. We stress again that in this situation a unitary PMNS matrix forbids the generation of a baryon asymmetry via leptogenesis. The correct value of the baryon asymmetry can be generated with values of  $A_{\mu\tau}^{\text{NU}}$  almost four orders of magnitude above the SM value. Note that we have plotted only positive values of  $Y_B$ . We also show in Fig. 1 the case of  $\theta_{13} = 0.1$ , in which case we have also varied  $\delta$ .

It turns out that  $|\eta_{\mu\tau}| \gtrsim 10^{-4}$  in order to generate a sufficient  $Y_B$ . Interestingly, these are values for which future facilities will be able to probe the non-standard CP phases [7]. In Fig. 2 we show  $|\eta_{\mu\tau}|$  against the baryon asymmetry of the Universe for the case of  $\theta_{13} = 0$  and everything else as for Fig. 1. We have also indicated certain values of  $|\eta_{\mu\tau}|$  which correspond to certain observable benchmark values of the branching ratio for the decay  $\tau \rightarrow \mu\gamma$ . Though this is a straightforward and rather simple application, it serves to illustrate nicely the rich phenomenology of unitarity violation.

## 4 Mass Limits from Leptogenesis

Finally, let us note that the upper limit on the decay asymmetry  $\varepsilon_1$  is also modified if the PMNS matrix is not unitary. A close numerical inspection of the situation reveals that for a lightest neutrino mass of 0.15 eV, one can exceed the upper bound on  $\varepsilon_1$  (obtained for a unitary PMNS matrix) as a function of  $\tilde{m}_1$  by roughly 30 %. Analytically, in the unitary case and with the parametrization of  $R$  from above one can choose [24]  $\theta_{12}^R = 0$  and  $\theta_{13}^R = \rho_{13} + i\sigma_{13} = \pi/4 + i\sigma_{13}$  with real  $\sigma_{13}$  to find that

$$\varepsilon_1 = \frac{3 M_1}{16 \pi v^2} (m_3 - m_1) \tanh 2\sigma_{13}$$

approaches the Davidson-Ibarra bound [25]  $\varepsilon_1^{\text{DI}} = \frac{3 M_1}{16 \pi v^2} (m_3 - m_1)$  in the limit of large  $\sigma_{13}$ . Ref. [26] obtained a more general limit as a function of  $m_1$ ,  $\tilde{m}_1$  and  $m_3$ :

$$|\varepsilon_1| \leq \frac{1}{2} \varepsilon_1^{\text{DI}} \sqrt{1 - ((1 - a) \tilde{m}_1 / (m_3 - m_1))^2} \sqrt{(1 + a)^2 - ((m_3 + m_1) / \tilde{m}_1)^2}, \quad (21)$$

with

$$a = 2 \operatorname{Re} \left( \frac{m_1 m_3}{\tilde{m}_1^2} \right)^{1/3} \left( -1 - i \sqrt{\frac{(m_1^2 + m_3^2 + \tilde{m}_1^2)^3}{27 m_1^2 m_3^2 \tilde{m}_1^2}} \right)^{1/3}. \quad (22)$$

Now we consider modifications of these limits in case of a non-unitary PMNS matrix. As usual, we consider only  $|\eta_{\mu\tau}| e^{i\phi_{\mu\tau}}$ , while choosing in  $U_0$  the angles to be  $\theta_{23} = \pi/4$ ,  $\theta_{13} = 0$ ,  $\theta_{12} = \sin^{-1} \sqrt{1/3}$  and all phases zero, and finally we find

$$\begin{aligned} \varepsilon_1 &\simeq \frac{3 M_1}{16 \pi v^2} (m_3 - m_1) \tanh 2\sigma_{13} \\ &+ \frac{M_1}{8 \pi v^2} |\eta_{\mu\tau}| (m_1 + 3 m_3) \cos \phi_{\mu\tau} \tanh 2\sigma_{13}. \end{aligned} \quad (23)$$

We have given here only the leading term proportional to  $|\eta_{\mu\tau}|$ . The ratio of this term to the zeroth order term is smaller than

$$\frac{\frac{1}{8} (m_1 + 3 m_3) |\eta_{\mu\tau}|}{\frac{3}{16} (m_3 - m_1)} \simeq \frac{16}{3} \frac{m_1^2}{\Delta m_A^2} |\eta_{\mu\tau}| \lesssim 0.25,$$

where we have used that for  $m_1 = 0.15$  eV the neutrinos are quasi-degenerate and  $m_3 - m_1 \simeq (m_3^2 - m_1^2) / (2 m_1)$ . This estimate almost fully explains the 30% effect seen in the numerical analysis. The reason for the comparably large contribution from the small term  $|\eta_{\mu\tau}|$  is that the zeroth order expression  $\varepsilon_1^{\text{DI}}$  goes to zero for  $m_3 \rightarrow m_1$ , whereas the terms proportional to  $|\eta_{\mu\tau}|$  do not. We show in Fig. 3 scatter plots of the decay asymmetry against  $\tilde{m}_1$ . The solid (blue) line in the left plot shows the exact upper bound from Eq. (21), the dashed (green) line shows the Davidson-Ibarra bound. In the right plot of Fig. 3 the ratio of  $\varepsilon_1$  to its maximal value Eq. (21) in case of a unitary mixing matrix is shown. We have chosen  $\theta_{12}^R = 0$ ,  $\theta_{13}^R = \rho_{13} + i\sigma_{13} = \pi/4 + i\sigma_{13}$ , a normal mass ordering with  $m_1 = 0.15$  eV,

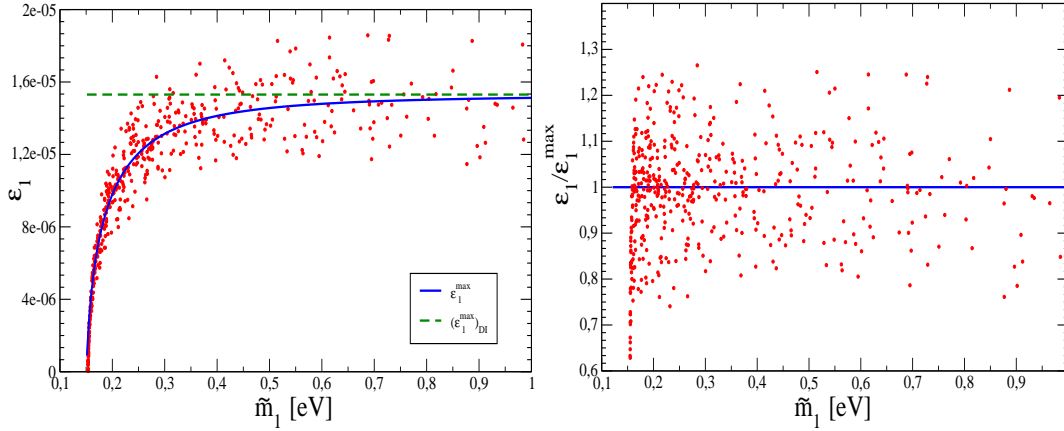


Figure 3: Scatter plot of the decay asymmetry  $\varepsilon_1$  against  $\tilde{m}_1$  (left) and the ratio of  $\varepsilon_1$  against its maximal value in case of a unitary PMNS matrix (right).

$\theta_{23} = \pi/4$ ,  $\theta_{13} = 0$ ,  $\theta_{12} = \sin^{-1} \sqrt{1/3}$  and all phases in  $U_0$  zero. As seen from the plots, the non-unitarity of the mixing matrix can violate the upper bound on  $\varepsilon_1$  by 30 %, as mentioned above.

From a limit on  $\varepsilon_1$  it is possible to get limits on the light and heavy neutrino masses from a detailed analysis of the wash-out [26, 27]. For instance, Ref. [26] found that a limit of  $m_1 \leq 0.15$  eV is implied, and Ref. [27] quotes  $M_1 \geq 4 \cdot 10^8$  GeV. We note here that these limits may get modified by the presence of non-unitarity, simply because the upper limit on  $\varepsilon_1$  is modified. A detailed analysis is beyond the scope of this letter, but we can estimate the effect. We note first that in contrast to the contribution to  $\varepsilon_1$ , the corrections from  $|\eta_{\mu\tau}|$  to the wash-out parameter  $\tilde{m}_1$  are not enhanced. The wash-out therefore proceeds to good precision in the same manner as for a unitary PMNS matrix and thus the limit on  $m_1$  is modified with good precision by the same amount as the limit on  $\varepsilon_1$  is modified. Nevertheless, the upper limit on the light neutrino mass, which is roughly proportional to  $|\varepsilon_1^{\max}|^{0.25}$  [28] is not substantially different than before, thus  $m_1 \lesssim 0.16$  eV instead of 0.15 eV. In addition, given the uncertainties [26] of such bounds, the modification due to unitarity violation is not of much significance. The lower limit on the lightest heavy neutrino is inversely proportional to  $|\varepsilon_1^{\max}|$ , if hierarchical light neutrinos are assumed. In this limit, however, the impact of non-unitarity on the decay asymmetry is suppressed by the smallness of the elements of  $\eta$ , and hence limits on  $M_1$  are basically not modified.

## 5 Summary

The possible non-unitarity of the lepton mixing matrix has several consequences in phenomenology. We have illustrated here in particular that the non-standard CP phases as induced by non-unitarity have the capacity to lead to observable effects in neutrino oscillation experiments as well as to a successful generation of the baryon asymmetry of the Universe via leptogenesis. This is true even for situations in which a unitary lepton

mixing matrix would lead to no leptogenesis at all. The usual “no connection” theorem of low and high energy CP violation in case of unflavored leptogenesis is thus avoided. Lepton Flavor Violation in charged lepton decays such as  $\tau \rightarrow \mu\gamma$  can at the same time be induced at an observable level. The neutrino mass constraints from leptogenesis are only slightly modified. We have discussed an extension of the see-saw mechanism which can incorporate this framework, but our results will apply also for other scenarios as long as the source of non-unitarity is decoupled from leptogenesis and from  $m_\nu = -m_D^T M_R^{-1} m_D$ .

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